Étale Cohomology Learning Seminar Syllabus

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Logistics

This is the syllabus of UIUC étale cohomology learning seminar in summer 2024, mainly following Milne's *Étale Cohomology* ([Mil80]) and Fu's *Étale Cohomology Theory* ([Fu11]). More details can be found online.

Topics

The list of topics below is adapted from [Mil12], taking [Mil80] and [Fu11] into account. Note that some topics may take multiple talks to finish, while others may take half of a talk's time, depending on speakers' choices.

Étale Morphism

1. Flat morphisms and Descent theory
   - [Fu11], chapter 1; [Mil80], chapter 1.2.
2. Étale topology, étale morphisms
   - [Mil80], chapter 1.3; [Fu11], chapter 2.1-2.4, 2.9; [Mil12], lecture 2.
3. Henselian rings
   - [Mil80], chapter 1.4; [Fu11], chapter 2.8; [Mil12], lecture 4.
4. Étale fundamental group
   - [Mil80], chapter 1.5; [Fu11], chapter 3; [Mil12], lecture 3.

* *Grothendieck topology, sites*

- [Mil12], lecture 5.

Sheaf Theory

5. Sheaves for étale topology
   - [Mil80], chapter 2.1; [Fu11], chapter 5.1-5.5; [Mil12], lecture 6.
6. Category of étale sheaves
   - [Mil80], chapter 2.2; [Fu11], chapter 5.1-5.5; [Mil12], lecture 7.

7. Direct and inverse images of sheaves
    - [Mil80], chapter 2.3; [Fu11], chapter 5.1-5.5; [Mil12], lecture 8.

Cohomology

* *Group cohomology and Galois cohomology*
    - [Fu11], chapter 4.1-4.3, 4.5.

8. Étale cohomology
    - [Mil80], chapter 3.1; [Fu11], chapter 5.6-5.7; [Mil12], lecture 9.
9. Čech cohomology
    - [Mil80], chapter 3.2; [Fu11], chapter 5.1, 5.6-5.7; [Mil12], lecture 10.
10. Comparison of topologies
    - [Mil80], chapter 3.3; [Fu11], chapter 5.6-5.7; [Mil12], lecture 9.
11. Principal homogeneous spaces
    - [Mil80], chapter 3.4; [Mil12], lecture 11.

Cohomology of curves and surfaces

12. Constructible sheaves
    - [Mil80], chapter 5.1; [Fu11], chapter 5.8-5.9; [Mil12], lecture 14, 17.
13. Cohomology of curves
    - [Mil80], chapter 5.2; [Fu11], chapter 7.2; [Mil12], lecture 14.

* *Cohomology of surfaces*
    - [Mil80], chapter 5.3.

The Fundamental Theorems

14. Cohomological dimension
    - [Mil80], chapter 6.1; [Fu11], chapter 4.4; [Mil12], lecture 15.
15. Purity and Gysin sequence
    - [Mil80], chapter 6.5; [Mil12], lecture 16.
16. Proper base-change theorem
    - [Mil80], chapter 6.2; [Fu11], chapter 7.3; [Mil12], lecture 17.
17. Higher direct image and Leray spectral sequence
    - [Mil80], chapter 6.3; [Fu11], chapter 5.6; [Mil12], lecture 12.
18. Cohomology with compact support

- [Mil80], chapter 6.3; [Fu11], chapter 7.4-7.5; [Mil12], lecture 18.

19. Finiteness theorem
    - [Mil80], chapter 6.2; [Fu11], chapter 7.8; [Mil12], lecture 19.
20. Smooth base-change theorem
    - [Mil80], chapter 6.4; [Fu11], chapter 7.7; [Mil12], lecture 20.

* *Adic formalism*

- [Fu11], chapter 10.1.

21. Künneth formula
    - [Mil80], chapter 6.8; [Fu11], chapter 7.4; [Mil12], lecture 22.
22. Fundamental class, cycle map, and Chern class
    - [Mil80], chapter 6.6, 6.9-6.10; [Mil12], lecture 23.

* Trace morphisms

- [Mil80], chapter 6.11; [Fu11], chapter 8.1-8.2; [Mil12], lecture 24.

23. Poincaré/Verdier duality
    - [Mil80], chapter 6.11; [Fu11], chapter 8.5-8.6; [Mil12], lecture 24.

* *Étale-analytic comparison: Artin-Grothendieck theorem*

- [Mil12], lecture 21.

24. Lefschetz fixed point formula and Grothendieck-Lefschetz trace formula
    - [Mil80], chapter 6.12; [Fu11], chapter 8.6, 10.2-10.4; [Mil12], lecture 25.

References

[FK13] Eberhard Freitag and Reinhardt Kiehl. *Etale cohomology and the Weil conjecture*, volume 13. Springer Science & Business Media, 2013.

[Fu11] Lei Fu. *Etale cohomology theory*, volume 13. World Scientific, 2011.

[Mil80] James S Milne. *Etale cohomology (PMS-33)*. Princeton university press, 1980.

[Mil12] James S Milne. Lectures on étale cohomology, 2012.

[Tam12] Günter Tamme. *Introduction to étale cohomology*. Springer Science & Business Media, 2012.